



Development of a Realistic Mathematics Teaching Module on Arithmetic Sequences and Series to Support the Mathematical Literacy of Vocational High School Students

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Abstract

The low mathematical literacy of Indonesian students remains a persistent issue, partly due to procedural and decontextualised teaching practices, including in arithmetic sequences and series at vocational high schools. This study aimed to develop a realistic mathematics teaching module on arithmetic sequences and series that is valid, practical, and effective in supporting the mathematical literacy of grade X students. The study used a Research and Development approach based on Plomp's model, consisting of preliminary research, development/prototyping, and assessment. The subjects were 22 grade X students of SMK Negeri 1 Amurang. Three expert validators assessed module validity, while eight students evaluated practicality in a small-group trial. The instruments included validation sheets, student response questionnaires, and a mathematical literacy test based on formulating, employing, and interpreting indicators. The results showed that the module was highly valid with a score of 86%, practical with a score of 81.8%, and effective, with a mean N-Gain of 0.84 and 86.36% classical completeness. The mean score increased from 17.73 in the pretest to 86.45 in the posttest. These findings indicate that the module effectively connects mathematical concepts with vocational contexts and strengthens students' mathematical literacy.

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Introduction

Education is a strategic instrument for building human resources capable of adapting to increasingly complex social, economic, and technological change. Within the contemporary educational paradigm, the success of learning is no longer measured solely by the procedural mastery of content, but rather by learners' capacity to understand, interpret, and use knowledge meaningfully in real life. One of the key competencies receiving global attention is mathematical literacy, which is regarded as an important foundation for developing logical and critical thinking and problem-solving abilities. Mathematical literacy is defined as an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts, including reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena (OECD, 2019). This view emphasises that mathematics learning should not focus only on computation, but also on conceptual understanding and its application to real situations as a provision for reflective citizens (Niss, 2003).

Various educational evaluation reports indicate that the mathematical literacy of Indonesian students remains at a concerning level. The results of the Programme for International Student Assessment (PISA) consistently show that most Indonesian students struggle to solve contextual mathematics problems that demand higher-order reasoning, with attainment well below the international average (Domu & Mangelep, 2020; OECD, 2019; Stacey, 2011). Students tend to be able to solve routine problems but encounter difficulties when faced with tasks that require mathematical modelling and the interpretation of real situations. A number of studies emphasise that these difficulties are not merely a matter of students' cognitive ability but are closely related to the types of tasks and contexts that are rarely practised in the classroom (Domu & Mangelep, 2024; Sáenz, 2009; Wijaya et al., 2014).

This condition is inseparable from mathematics teaching practices that remain dominated by conventional approaches. Learning is often presented in an abstract, symbolic manner, focused on procedures for solving problems, so that students have little opportunity to construct meaningful conceptual understanding. As a result, mathematics is perceived as a difficult, rigid subject detached from everyday life. Skemp (1976) distinguishes between instrumental and relational understanding and argues that learning that emphasises only instrumental understanding produces students who can use formulas but do not grasp the meaning behind the procedures. This dominance of algorithmic instruction partly explains students' low engagement and their difficulty in applying mathematical concepts to new contexts.

This problem becomes even more complex at the level of Vocational High Schools (Sekolah Menengah Kejuruan, SMK). SMK have a distinctive character, namely preparing graduates who are ready to enter the workforce and able to apply academic knowledge in practical and professional contexts. Mathematics learning at SMK should therefore be designed contextually and applicatively in order to be relevant to students' needs. In reality, however, mathematics learning at SMK still largely uses generic teaching materials that do not accommodate vocational contexts or students' real experiences. Yet mathematical literacy among SMK students plays a strategic role because it relates directly to the ability to understand vocational problems such as cost calculation, data analysis, and number-based decision making (Moroki et al., 2025; Ni'mah et al., 2017).

One important component of learning is the teaching module. A module functions as an independent learning resource designed systematically to help students achieve particular competencies. A good module not only contains material and practice problems, but also facilitates students' thinking processes, provides meaningful learning experiences, and encourages active engagement. Unfortunately, the mathematics modules used at SMK generally still emphasise the presentation of material and procedural practice without linking concepts to the real-life contexts of students. The topic of arithmetic sequences and series actually has great potential for contextual development because it is frequently encountered in daily life, such as in instalment calculations, savings interest, production growth, and financial planning. In practice, however, this topic is often presented merely as formulas and algebraic manipulation, so that students find it difficult to grasp its meaning and usefulness.

A learning approach considered capable of bridging the gap between mathematical concepts and real-life contexts is Realistic Mathematics Education (RME). This approach is grounded in the view that mathematics is a human activity (Freudenthal, 1991). In RME, learning begins from realistic contextual problems, and students are then guided to reinvent mathematical concepts through horizontal and vertical mathematisation (Gravemeijer & Doorman, 1999). In Indonesia, RME has been adapted into Pendidikan Matematika Realistik Indonesia (PMRI) and has developed as a movement to reform mathematics learning since the early 2000s (Sembiring et al., 2008). This approach aligns with the goal of developing mathematical literacy because it emphasises understanding, reasoning, and the application of mathematics in real contexts, as shown by various studies on its effectiveness for conceptual understanding and modelling (Domu & Mangelep, 2025; Juandi et al., 2021; Revina & Leung, 2019).

Although many studies have examined the effectiveness of RME in supporting conceptual understanding and problem solving, research that specifically integrates mathematical literacy indicators as formulated by OECD (2019) into module design on the topic of sequences and series in the SMK context remains very limited. Most studies focus on the implementation of the approach in classroom learning without producing systematic teaching materials that can be used sustainably, or they develop RME devices on other topics such as social arithmetic and algebra (Mangelep, 2025; Sari & Sari, 2019). The development of modules that explicitly facilitate the processes of formulating, employing, and interpreting mathematics in the vocational contexts of SMK students is still rarely found. This situation reveals both a conceptual and a practical gap that calls for development research grounded in a systematic model.

Preliminary observations and interviews with mathematics teachers at SMK Negeri 1 Amurang reinforce this urgency. Learning on the topic of arithmetic sequences and series is still dominated by lecturing and routine practice; students tend to be passive and struggle with contextual problems, so that the average test score on this topic in the previous academic year was only 62.4, below the Minimum Competency Criterion (MCC) of 75. Moreover, no RME-based module designed from an analysis of local students' needs was available. Based on this gap, the present study focuses on developing a realistic mathematics teaching module on arithmetic sequences and series to support the mathematical literacy of grade X students. The research questions posed are how to develop a module that meets the criteria of validity, practicality, and effectiveness. The findings are expected to make a theoretical contribution to the development of contextual mathematics teaching materials as well as a practical contribution for teachers in delivering more meaningful learning at the SMK level.

Method

This study is Research and Development (R&D) employing the Plomp model (2013). This model was chosen because it is systematic and based on formative evaluation, allowing the product to be developed iteratively until a valid, practical, and effective module is obtained. The study falls into the category of design research of the development study type, aiming to produce an educational product in the form of a realistic mathematics teaching module on arithmetic sequences and series that is suitable for use in learning. The Plomp model comprises three phases: preliminary research, development/prototyping phase, and assessment phase.

In the preliminary research phase, a needs analysis was conducted through observation of learning and interviews with mathematics teachers, a curriculum analysis of the Learning Outcomes and Learning Objective Flow in the Merdeka Curriculum, an analysis of student characteristics, and a concept analysis of arithmetic sequences and series and their relationship to mathematical literacy. The output of this phase was the initial product specification and the design of the RME-based module. In the development/prototyping phase, Prototype I was constructed, integrating RME principles and mathematical-literacy-based problems, and was then refined through layered formative evaluation: self-evaluation by the researcher, expert review by validators, one-to-one evaluation with three students of high, medium, and low ability, and small-group evaluation with eight students. Revisions were made at each stage until the final prototype was obtained.

The research subjects were three expert validators (two content experts and one instructional design expert) and grade X students of SMK Negeri 1 Amurang. The class was selected purposively, considering the suitability of the material and the readiness of the teacher and students. The object of the study was the realistic mathematics teaching module on arithmetic sequences and series. In the assessment phase, an effectiveness test was conducted on 22 students using a One Group Pretest-Posttest design, namely O1 X O2, with O1 as the mathematical literacy pretest, X as learning using the module, and O2 as the posttest.

Data collection techniques comprised observation, interviews, validation questionnaires, practicality questionnaires, and a mathematical literacy test. The research instruments consisted of an expert validation sheet assessing content feasibility, presentation feasibility, language feasibility, conformity with RME principles, and the attainment of mathematical literacy indicators; a practicality questionnaire assessing ease of use, clarity of instructions, ease of understanding the material, and student engagement; a mathematical literacy test constructed on the basis of the formulating, employing, and interpreting indicators; and a learning-implementation observation sheet. The validation sheet and questionnaires used a Likert scale.

Data were analysed descriptively. Validity was calculated as the percentage $V = (\Sigma X / \Sigma X_{\max}) \times 100\%$, with criteria of 81–100% highly valid, 61–80% valid, and 41–60% moderately valid. Practicality was calculated as the percentage $P = (\Sigma X / \Sigma X_{\max}) \times 100\%$, with criteria of $\geq 85\%$ highly practical, 70–84% practical, and $< 70\%$ less practical. Effectiveness was analysed through N-Gain, $g = (\text{Spost} - \text{Spre}) / (\text{Smax} - \text{Spre})$, with categories of $g \geq 0.70$ high, $0.30 \leq g < 0.70$ medium, and $g < 0.30$ low, as well as classical completeness $CC = (nt / N) \times 100\%$. The module was declared effective if the mean N-Gain was in the medium or high category and the classical completeness reached $\geq 75\%$.

Results and Discussions

1. Preliminary Research Results

The preliminary research phase was carried out to identify the fundamental problems hindering learning. Data were obtained from interviews with teachers and students and from the distribution of a needs-analysis questionnaire to grade X students. The results of the needs analysis are summarised in Table 1.

Table 1. Summary of the Needs Analysis Results

Analysis aspect	Data / Findings	Indicated module need
Classroom observation	Teaching materials lack context; students struggle with reasoning problems	Activity-based module, RME integration
Teacher interview	Generic teaching materials; literacy not yet supported	SMK context-based module, literacy-based problems
Student learning outcomes	Mean score 62.4 (MCC 75)	Module must strengthen conceptual understanding & practice
Student characteristics	Varied prior ability; high interest in practice	Adaptive & interactive module
Curriculum analysis	Outcomes align with Merdeka Curriculum; Pancasila Student Profile relevant	Module aligned with curriculum & 21st-century competencies
Concept analysis	Arithmetic sequences & series; link to literacy	Module facilitates Formulating-Employing-Interpreting

Based on Table 1, mathematics learning on the topic of arithmetic sequences and series is still dominated by lecturing and routine practice, while the teaching materials used are generic and do not yet support mathematical literacy. The students' average learning outcome on this topic was only 62.4, still below the MCC of 75. The curriculum analysis showed that the Phase E Learning Outcomes require students to understand patterns, determine the n th term, determine the sum of the first n terms, and solve contextual problems—demands that align with the development of mathematical literacy. The analysis of student characteristics revealed heterogeneous prior abilities, yet with a higher interest in learning linked to real life.

The concept analysis identified two main concepts, namely arithmetic sequences (covering the first term, common difference, n th term, and regularity pattern) and arithmetic series (covering the number of terms, the n th term, and the sum of the first n terms). Understanding of the arithmetic sequence is positioned as a prerequisite for the arithmetic series, because a series concerns the summation of the terms of a sequence. The two concepts are linked to contextual problems in order to facilitate the three processes of mathematical literacy, as depicted in the concept map in Figure 1. The material is arranged from the simplest to the more complex, starting from recognising patterns, determining the first term and common difference, determining the n th term, understanding the concept of a series, calculating the sum of the first n terms, and interpreting solutions in real-life contexts such as savings, instalments, and production patterns.

The concept map in Figure 1 shows that arithmetic sequences and series are not treated as two separate topics but are woven into a single flow towards contextual problems, which then become the

entry point for the three processes of mathematical literacy—formulating, employing, and interpreting. This structure forms the basis for arranging the material flow, learning activities, and mathematical-literacy problems in the module, so that each activity can be mapped explicitly onto the targeted literacy indicator.

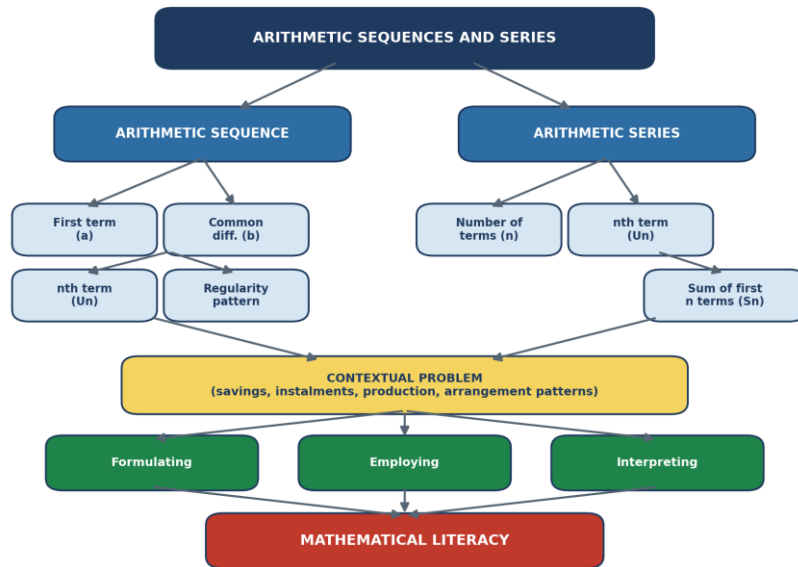


Figure 1. Concept map of arithmetic sequences and series and its link to mathematical literacy

2. Development/Prototyping Phase Results

In the development/prototyping phase, the researcher constructed the initial module design (Prototype I) based on the results of the four analyses in the previous phase. The module structure consists of a cover page, preface, table of contents, module identity, learning outcomes, learning objectives, usage instructions, RME-based learning activities, mathematical literacy assessment, summary, and references, and is equipped with Student Worksheets in each activity. The material is presented systematically, beginning with a contextual problem used as the starting point for discovering the concepts of arithmetic sequences and series through mathematisation, followed by the development of a mathematical model, discussion of solution strategies, and the drawing of conclusions.

Prototype I was validated by three validators, namely two content experts and one instructional design expert. The validators' assessments generally took the form of improvement notes that were followed up iteratively. The main suggestions included improving the layout to be neater and more consistent and inserting the Student Worksheets; sharpening the examples to be more realistic and strengthening critical-thinking activities; formulating learning objectives that are more specific and measurable in accordance with the formulating–employing–interpreting indicators; and refining the activity flow so that the RME syntax is complete, interactive, and equipped with reflection and mathematical literacy assessment. As an illustration, a display initially dominated by text with inconsistent formula and table formats was improved to be neater with visuals supporting understanding; several examples that were initially unrealistic were replaced with contexts closer to students' lives; and an activity flow that was initially linear with passive student activity was changed into an interactive RME syntax.

In addition to expert review, the module also underwent a one-to-one evaluation with three students of high, medium, and low ability to identify comprehension difficulties and the clarity of instructions, followed by a small-group evaluation. Input from these stages was used to refine the readability of the instructions and the adequacy of the working space on the Student Worksheets. After all revisions, the validators stated that the module conformed to RME principles and supported the development of mathematical literacy, so that the module was declared suitable for trial.

The validation results for the five assessment aspects are summarised in Table 2. The content feasibility aspect obtained 85%, presentation feasibility 77.7%, language feasibility 88.3%, conformity with RME principles 93.3%, and attainment of mathematical literacy indicators 85.5%. Overall, the average percentage of module validity reached 86%, in the highly valid category. The conformity with RME principles aspect obtained the highest percentage (93.3%), indicating that the module consistently applies the characteristics of RME such as the use of real contexts and self-developed models. The presentation aspect obtained the lowest percentage (77.7%) but remained in the valid category and became a focus of improvement during the revision stage, as reflected in the refinement of the layout and formatting consistency. The consistency of the three validators' assessments—with average scores of 16.6, 17.2, and 17.8 indicates adequate reliability of the assessment.

Table 2. Module Validation Results by Three Validators

Assessed aspect	Score obtained	Max score	Percentage	Category
Content feasibility	51	60	85.0%	Highly valid
Presentation feasibility	35	45	77.7%	Valid
Language feasibility	53	60	88.3%	Highly valid
Conformity with RME principles	42	45	93.3%	Highly valid
Attainment of literacy indicators	77	90	85.5%	Highly valid
Average	51.6	60	86.0%	Highly valid

3. Assessment Phase Results

a. Module Practicality

In the assessment phase, the practicality test was conducted with eight grade X students through a response questionnaire. The complete results are presented in Table 3. The ease-of-use aspect obtained 83.3%, clarity of usage instructions 82.5%, ease of understanding the material 80%, and student engagement in learning 81.6%. The average percentage of practicality reached 81.8%, in the practical category. The four aspects obtained relatively balanced percentages in the 80–83% range, indicating that no component of the module stood out as problematic from the users' viewpoint. The ease-of-use aspect obtained the highest value, reflecting the success of the layout improvements and step-by-step instructions resulting from the revision. The ease-of-understanding aspect obtained the lowest value, which is understandable given the heterogeneity of students' prior abilities as identified in the preliminary research phase. Overall, these results show that the module is easy to use, easy to understand, and helps students study the topic of arithmetic sequences and series independently.

Table 3. Student Response Questionnaire Results on Module Practicality

Assessed aspect	Score obtained	Max score	Percentage
Ease of using the module	100	120	83.3%
Clarity of usage instructions	99	120	82.5%

Ease of understanding the material	96	120	80.0%
Student engagement in learning	98	120	81.6%
Average	98.25	120	81.8% (Practical)

b. Module Effectiveness

The effectiveness test was conducted with 22 students through a mathematical literacy pretest and posttest. The mean pretest score of 17.73 increased to 86.45 in the posttest, as shown in Figure 2. The low pretest scores with most students scoring below 40 confirm the needs-analysis finding that students were not yet accustomed to problems requiring the processes of formulating, employing, and interpreting. After learning with the module, almost all students experienced a sharp increase, with the majority of posttest scores exceeding the MCC of 75. The N-Gain analysis yielded a mean of 0.84, in the high category. Of the 22 students, 18 obtained a high-category N-Gain and 4 a medium category, with none in the low category. A summary of the N-Gain distribution is presented in Table 4.

The pattern of improvement in Figure 2 shows that the module's effect was relatively even across prior-ability levels: students with very low pretest scores (e.g. 2-9) and those with relatively higher scores (e.g. 34-42) alike attained high posttest scores. This indicates that the context-based activities in the module are accessible to students with diverse ability backgrounds. The four students who obtained a medium-category N-Gain were generally those whose posttest scores were around or below the MCC, providing a note for further mentoring.

The classical completeness analysis in Table 5 shows that 19 of 22 students (86.36%) achieved scores above the MCC, while 3 students (13.64%) had not yet achieved mastery. Because the mean N-Gain is in the high category and the classical completeness exceeds the 75% threshold, the developed module is declared to meet the effective criterion. Thus, the three product-quality criteria validity (86%, highly valid), practicality (81.8%, practical), and effectiveness (N-Gain 0.84 and completeness 86.36%) have been met, so that the realistic mathematics teaching module on arithmetic sequences and series is suitable for use in supporting the mathematical literacy of grade X SMK students. It should be noted that this effectiveness was obtained through a one-group design without a comparison class, so that its interpretation focuses on the magnitude of within-group improvement rather than on between-group comparison.

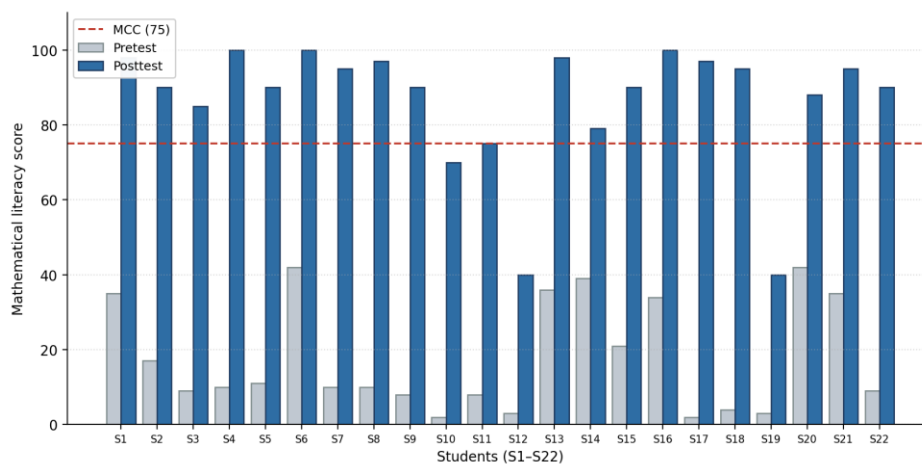


Figure 2. Comparison of pretest and posttest mathematical literacy scores of 22 students (dashed line = MCC 75)

Table 4. Distribution of N-Gain Categories of Students' Mathematical Literacy

N-Gain category	Range	Number of students	Percentage
High	$g \geq 0.70$	18	81.82%
Medium	$0.30 \leq g < 0.70$	4	18.18%
Low	$g < 0.30$	0	0.00%
Mean N-Gain	0.84 (High)	22	100%

Table 5. Students' Classical Completeness Results

Criterion	Number of students	Percentage
Achieved (score ≥ 75)	19	86.36%
Not yet achieved (score < 75)	3	13.64%
Total	22	100%

4. Discussion

The research results show that the developed realistic mathematics teaching module on arithmetic sequences and series meets the three product-quality criteria, namely highly valid (86%), practical (81.8%), and effective (N-Gain 0.84; classical completeness 86.36%). These attainments simultaneously answer the three research questions and confirm that the module can support the strengthening of SMK students' mathematical literacy. In this section, the three attainments are discussed in light of the theories of Realistic Mathematics Education and mathematical literacy, and are compared with the findings of previous research.

In general, these three attainments do not stand alone but reinforce one another. High validity ensures that the module design rests on a solid theoretical foundation; practicality ensures that this design can be translated into an accessible learning experience; and effectiveness demonstrates that this learning experience has an impact on the targeted ability. This interrelationship is consistent with the logic of development research, in which product quality is built in layers through formative evaluation before reaching the testing of impact (Plomp, 2013).

a. Module Validity

The module's validity, which reached the highly valid category, shows that the product is aligned with theoretical foundations and consistent among its components. The conformity with RME principles aspect obtained the highest percentage (93.3%), indicating that the characteristics of RME the use of real contexts, mathematical models, student contribution, interactivity, and interconnection have been integrated coherently. This finding is consistent with Freudenthal's (1991) notion that mathematics learning must begin from contextual situations so that concepts can be discovered meaningfully. The revision process, which emphasised the restructuring of activity steps, the sharpening of realistic examples, and the addition of visual illustrations, strengthened the principle of self-developed models, that is, a bridge from informal to formal mathematics (Gravemeijer, 1994).

The presentation aspect, which obtained the lowest percentage (77.7%), shows that the quality of visual representation and the consistency of format are the components that most demand refinement in module development. This aligns with development research on RME-based devices that found readability and the structure of presentation often to be critical points in early prototypes (Sari & Sari, 2019). The consistency of the three validators' assessments strengthens confidence in the results, and the entire validation process reflects the advantage of the layered formative evaluation in the Plomp model, which allows weaknesses to be identified and corrected before the field test (Plomp, 2013).

Thus, the module's validity relates not only to the completeness of content, but also to the integration of RME principles and its capacity to facilitate mathematical literacy indicators.

The high score on the attainment of mathematical literacy indicators (85.5%) is an important marker that the module does not merely add context to problems, but designs activities that deliberately target the processes of formulating, employing, and interpreting. This is consistent with the assertions of OECD (2019) and Niss (2003) that mathematical literacy requires an explicit connection between mathematical knowledge and real situations. Positioning the contextual problem as the point of departure rather than as a closing element after the formula is given reflects didactical phenomenology, namely the selection of mathematically rich phenomena as a source of concept construction, so that the module's validity has a strong didactic foundation.

b. Module Practicality

The module's practicality, which is in the practical category, shows that the module is easy to use by both students and teachers. The four practicality aspects obtained balanced percentages in the 80–83% range, with ease of use as the highest aspect. This attainment is the fruit of improvements to the layout, the addition of information blocks, and step-by-step instructions that enable students to learn independently. This finding aligns with the principle that a practical module must facilitate independent learning, have clear instructions, and allow students to adjust the pace of learning to their respective abilities.

These results also confirm the findings of previous RME module development research, which concluded that an interactive structure and easy navigation increase practicality for both students and teachers. In the SMK context, practicality becomes even more important because the module is expected to be usable for student-centred learning and adjustable to classroom needs. The ease-of-understanding aspect, which obtained the lowest value, is a reminder that the heterogeneity of students' prior abilities still needs to be accommodated, for example through the provision of tiered examples or additional scaffolding. Nonetheless, overall the module has met the requirements of practicality for use in actual learning.

It is important to note that practicality in this study was measured through the responses of eight students in a small-group evaluation, so that it represents the student-user perspective more than the teacher-user perspective. Triangulation with teacher responses and learning-implementation data in a wider trial would strengthen the practicality claim, particularly regarding the ease with which teachers manage the RME syntax, which requires the facilitation of discussion and reflection. This is relevant because the success of RME in the classroom is strongly influenced by the teacher's ability to pose questions that encourage mathematisation, rather than merely conveying procedures.

c. Module Effectiveness

The module's effectiveness is reflected in the increase in mathematical literacy scores from a mean of 17.73 to 86.45, with an N-Gain of 0.84 in the high category and a classical completeness of 86.36%. This large increase shows that the module activities—involving the observation of patterns, the construction of tables, modelling, group discussion, and reflection—can facilitate students in formulating problems, employing the concepts of sequences and series, and interpreting results in real contexts. This pattern aligns with the principle of progressive mathematisation, in which students do not merely receive formulas but build their own solution strategies through guided reinvention before being directed to the formal concept (Freudenthal, 1991; Gravemeijer, 1994).

This finding strengthens Stacey's (2011) argument that learning involving the processes of formulating, employing, and interpreting mathematics in real contexts significantly supports mathematical literacy. The results are also consistent with studies showing that students' difficulties with

context-based tasks can be reduced when learning explicitly trains the transformation of real problems into mathematical models and the reinterpretation of results (Sáenz, 2009; Wijaya et al., 2014). More broadly, the magnitude of this increase adds empirical evidence for the effectiveness of the RME approach that has been reported meta-analytically over the past two decades in Indonesia (Juandi et al., 2022), as well as for the PMRI movement as a reform of mathematics learning (Sembiring et al., 2008).

The relatively even distribution of improvement across prior-ability levels shows that the context-based module is accessible to students with diverse ability backgrounds, an important characteristic for heterogeneous SMK classes. This is relevant to the finding that the mathematical literacy of vocational students can be developed through tasks that stimulate logical and numerical reasoning (Ni'mah et al., 2017). It can be interpreted that the very low pretest scores reflect students' habit of solving routine problems, while the surge in posttest scores marks the opening of access to higher-order thinking processes when students are given the opportunity to formulate and interpret contextual problems. In other words, the increase is not merely quantitative but indicates a shift in the quality of cognitive engagement from instrumental towards relational understanding, as distinguished by Skemp (1976).

Nonetheless, there are methodological limitations that should be noted. The research design used was a one-group pretest-posttest without a control class, so that it is difficult to separate the influence of the module from other external factors such as testing effects and maturation. Therefore, the effectiveness attained in this study should be interpreted as an indication of potential rather than strong causal evidence. In addition, the effectiveness analysis was conducted descriptively through N-Gain and classical completeness without an inferential test, so that the statistical significance of the increase has not been formally tested.

d. Connection with the Plomp Model, Implications, and Limitations

The success of module development is inseparable from the systematic application of the Plomp model. Each phase from initial investigation, the design and construction of the prototype, to assessment was followed by iterations of improvement based on input from validators and student responses. This process affirms the principle of research and development that emphasises the importance of iteration and continuous improvement, and demonstrates the alignment of the Plomp methodology with the principles of guided reinvention and progressive mathematisation in RME (Gravemeijer, 1994; Plomp, 2013). The prototyping phase emphasises the exploration of real problems before being directed to formal formulas, while the assessment phase emphasises reflection and refinement—both aligned with the principle of developing self-developed models.

The practical implication of this finding is the availability of an alternative contextual teaching material that SMK teachers can use to support student-centred learning in accordance with the demands of the Merdeka Curriculum, while also serving as a reference for constructing literacy-based problems and activities. Theoretically, this study enriches the study of RME-based teaching material development by linking it explicitly to the three processes of OECD mathematical literacy on the topic of arithmetic sequences and series in vocational contexts a gap that had not previously been widely filled. Unlike some RME studies that stop at classroom implementation, this study produces a systematic product that can be used sustainably.

Nonetheless, the generalisation of the findings should be made cautiously given that the subjects were limited to one class at one school. Further research is advised to use a quasi-experimental design with a control class and inferential statistical tests—such as a paired t-test or the Wilcoxon test preceded by a normality test—so that the module's effectiveness can be analysed with greater statistical power and more valid interpretation. In addition, the successful implementation of RME is known to depend on adaptation to the local classroom culture (Revina & Leung, 2019), so that trials in different school

contexts and vocational programmes would strengthen the transferability of the module. The development of a digital version or e-module also has the potential to broaden access and interactivity, in line with the contemporary direction of using technology in mathematics learning.

The novelty of this study lies in the integration of three elements that have rarely been combined simultaneously, namely the RME approach, the OECD mathematical literacy indicators, and the vocational context on the topic of arithmetic sequences and series, packaged in a module product whose quality has been tested through the Plomp model. Thus, this study not only adds empirical evidence regarding the effectiveness of RME, but also contributes a concrete learning artefact that can be adopted. Such a contribution is important for strengthening the ecosystem of contextual teaching materials at the SMK level, which demands a close connection between mathematics learning and the needs of the world of work.

Conclusion

Based on the development research conducted, it can be concluded that the realistic mathematics teaching module on arithmetic sequences and series for supporting the mathematical literacy of grade X SMK students has met the three product-quality criteria. The module met the highly valid criterion with a percentage of 86%, the practical criterion with a percentage of 81.8%, and the effective criterion with a mean N-Gain of 0.84 in the high category and a classical completeness of 86.36%. The mean mathematical literacy score increased from 17.73 in the pretest to 86.45 in the posttest. These findings indicate that an RME-based module developed through the layered formative evaluation of the Plomp model can bridge mathematical concepts with students' vocational contexts while supporting the strengthening of mathematical literacy.

This study has limitations in its one-group pretest-posttest design without a control class and in its descriptive effectiveness analysis. It is therefore recommended that further research employ a quasi-experimental design with inferential statistical tests, expand the subjects to different school contexts and vocational programmes, and develop a digital version of the module to broaden access and interactivity.

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